KRASNOSEL'SKIY, M.A.

Krasnosel'skii, M. A. Two problems. Uspehi Matem. Nauk (N.S.) 6, no. 1(41), 162-163 (1951). (Russian)

The following problem is formulated: Let L denote a continuous mapping of the sphere S^n into itself, without fixed points. Let Φx be a continuous vector field on a closed $(n+1)$-cell Δ^{n+1} bounded by S^n with no zero-vectors on S^n and let this field satisfy the condition that the vectors Φx and Lx are never similarly directed for any point x of S^n. Show that Φ has zero-vectors in the cell Δ^{n+1}. The author discusses the theorem of Borsuk on which this problem is patterned. It is also stated that the problem above is related to one by Lusternik, only the two-dimensional case of which has so far been solved [Lusternik, Mat. Sbornik N.S. 16(58), 124-128 (1945); these Rev. 7, 136]. *L. Zippin*

Source: Mathematical Reviews, Vol 13 No. 5

KRASNOSEL'SKIY, M. A.

Krasnosel'skiĭ, M. A. Operators with monotonic minorants. Doklady Akad. Nauk SSSR (N.S.) 76, 481-484 (1951). (Russian)

Krein and Rutman [Uspehi Matem. Nauk (N.S.) 3, no. 1(23), 3-95 (1948) = Amer. Math. Soc. Translation no. 26 (1950); these Rev. 10, 256; 12, 341] have established existence theorems for characteristic vectors of certain (linear and nonlinear) operators A in a Banach space E that are positive and monotone with respect to the partial ordering $\geq$ introduced in E by a fixed cone $K \subset E$. In this paper a few similar results concerning nonmonotone operators are communicated. The completely continuous operator B is said to have a monotonic minorant A if $Bx \geq Ax$ for all $x \in K$, where A is an operator as mentioned above. It is stated that such an operator B has characteristic vectors of arbitrary norm in the cone K; moreover these characteristic vectors form a continuous branch F, i.e. F has a nonvoid intersection with the boundary of every open set containing the vertex of K. Various applications to the characteristic values problem for nonlinear integral equations are indicated.

[illegible] ([illegible] Inst.)

Source: Mathematical Reviews, Vol. 12 No. 6

KRASNOSELSKY, M.

Krasnosel'skiĭ, M. A. On the continuity of the operator $f(x, u(x))$. Doklady Akad. Nauk SSSR (N.S.) 73, [illegible] (1950). (Russian)

Let $f(x, u)$ be defined for all real u and for $x \in G$, a measurable set, and let $f(x, u)$ be continuous for almost all fixed $x \in G$ and [illegible] the continuity of the nonlinear operator $f(x, u(x))$ [illegible] under the hypothesis that G is of finite measure [illegible] by the author and some others [illegible] Doklady (N.S.) 73, 13–15 (1950); these Rev. 12, [illegible]. In this paper it is assumed that G is of infinite measure, and it is proved that if f transforms every function of [illegible] then f is continuous (in the obvious [illegible]) and bounded on every sphere. The author observes [illegible] Vainberg [ibid. 73, 253–255 (1950); these Rev. 12, [illegible]] to the effect that f is continuous if compact [illegible] f can be compact only if constant. [illegible] (Lafayette, Ind.)

Source: Mathematical Reviews, Vol. 12 No. 10

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Operators, Vector 21 Jul 51

"Problem Concerning the Points of Bifurcation,"
M. A. Krasnosel'skiy, Inst of Math, Acad Sci USSR

"Dok Ak Nauk SSSR" Vol LXXIX, No 3, pp 389-392

The number L (in eq F = LAF, where A is a continuous operator operating in a real Banach space E and satisfying the condition Aθ = θ (θ is a zero of E); F is an eigenvector of A) is called a point of bifurcation of operator A if for any pos nonzero e there exist in the interval (L-e,L+e) eigenvalues of the operator A to which correspond eigenfunctions with norm as small as desired. Submitted by Acad A. N. Kolmogorov 16 May 51.

211T63

USSR/Mathematics - Nonlinear Functionals 1 Dec 51

"Theory of Orlicz's Space," M. A. Krasnosel'skiy, Ya. B. Rutitskiy

"Dok Ak Nauk SSSR" Vol LXXXI, No 4, pp 497-500

Application of general methods and of theorems on nonlinear functional analysis to study of concrete classes of nonlinear operators is effected with aid of various concrete Banach spaces. Study of many operators with essentially nonpolynominal nonlinearities cannot be conducted with aid of spaces ordinarily applied (C, L_p). Authors

202T67

USSR/Mathematics - Nonlinear Functionals (Contd) 1 Dec 51

propose certain assumptions relating to the theory of L_{Φ} spaces (Orlicz spaces) for the case where the function $\Phi(u)$ does not satisfy the Δ_2-condition. (Cf. W. Orlicz, Bull international de l'Acad Pol, ser A, Cracovie, 1932.) Submitted by Acad A. N. Kolmogorov 25 Sep 51.

202T67

KRASNOSEL'SKIY, M.A.

KRASNOSEL'SKIY, M.O.

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Krasnosel's'kiĭ, M. O. Approximate computation of characteristic values and functions of perturbed operators. Dopovidi Akad. Nauk Ukrain. RSR 1952, 155–160 (1952). (Ukrainian. Russian summary)

Suppose A is a completely continuous operator in a real Hilbert space H, $A\varphi_0=\lambda_0\varphi_0$, $\lambda_0>0$, $\|\varphi_0\|=1$. The operator B defined by $B\psi=\psi-(\psi,\varphi_0)\varphi_0-\lambda_0^{-1}A\psi$ has a bounded inverse. Every eigenvector of the perturbed operator $K=A+D$, where D is a linear operator of sufficiently small bound, can be found as a solution of the nonlinear equation (a) $\|\psi\|K\psi-\psi=0$, the corresponding eigenvalue being $1/\|\psi\|$. The author shows that the sequence

$$\psi_0=\lambda_0^{-1}\varphi_0,\quad \psi_n=\psi_{n-1}+B^{-1}(\|\psi_{n-1}\|K\psi_{n-1}-\psi_{n-1})$$

converges to a solution of (a) provided $\|D\|<\delta$ where δ is a certain function of λ_0, $\|B^{-1}\|$, $\|B^{-1}A\|$. The advantages of this seemingly laborious procedure over others of established use are not discussed. *M. Golomb* (Lafayette, Ind.).

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Nonlinear Iteration Jan-Mar 52

"Iterative Process with Minimum Residual," M. A. Krasnosel'skiy and S. G. Kreyn

Ukrain Mat Zhur, Vol 4, No 1, pp 104-105

(Report given at 23 Oct 51 session of Sci Council of Inst of Math, Acad Sci Uk SSR.) Problem of approx soln of system of eqs $Bx=b$ (B positive definite square n-matrix, b known n-vector, x desired n-vector) can be solved by finding approx soln as close as possible to exact soln or residual-vector $\Delta_n = Bx_n - b$ as small as possible. Authors propose method of max decrease of residual, namely nonlinear iterative process $x_{n+1}=x_n-\Delta_n\cdot(B\Delta_n,\Delta_n)/(B\Delta_n,B\Delta_n)$, converging as rapidly as geometrical progression $r=(M-m)/(M+m)$ (M,m greatest and least characteristics of B). Compare this process with ordinary iterative process and 'steepest descent' developed by L.V. Kantorovich. Also studied nonlinear operator $Kf=f-(Bf,f)Bf/(Bf,Bf)$.

KRASNOSEL'SKIY, M.A.

Krasnosel'skiĭ, M. A. On the estimation of the number of critical points of functionals. Uspehi Matem. Nauk (N.S.) 7, no. 2(48), 157–164 (1952). (Russian)

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Let there be given on a connected manifold K a continuous involution A without invariant points, i.e., a continuous operator on K to K for which $Ax \neq x$ and $A^2x = x$. A set of the first kind on K is a compact set none of whose components contains both x and Ax. A set of kind n on K is a set each of whose compact parts may be divided into n sets of the first kind and for which some compact part may not be divided into $(n-1)$ sets of the first kind. This concept is closely related to that of category. The author illustrates its use to obtain a theorem of Lyusternik on the number of critical points of an even nonnegative weakly continuous functional on the sphere S of Hilbert space, and related results, without reference to the properties of projective space. He generalizes this concept to manifolds on which there are given periodic transformations of period r.

J. M. Danskin (Santa Monica, Calif.).

MATHEMATICAL REVIEWS (Unclassified)
Vol. 14, No. 1, January 1953, pp. 1-120

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Iteration Process, Approximation   Jul/Aug 52

"Note on the Distribution of Errors During the Solution of a System of Linear Equations by an Iteration Process," M. A. Krasnosel'skiy, S. G. Kreyn

"Uspekh Matemat Nauk" Vol VII, No 4 (50), pp 157-161

The purpose of the present note is to refute the hypothesis that the most probable errors are always considerably less then the max errors. As it

225T64

turns out, the max errors are the most probable errors. Considers the recurrent formula $x_{m+1} = Ax_m + b$, where A is a matrix.

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KRASNOSEL'SKIY,

Mathematical Reviews
Vol. 14 No. 11
Dec. 1953
Numerical and Graphical
Methods

Lopšic, A. M. An extremal theorem for a hyperellipsoid and its application to the solution of a system of linear algebraic equations. Trudy Sem. Vektor. Tenzor. Analizu 9, 183–197 (1952). (Russian)

Let A be a symmetric positive definite affinor [= matrix] whose largest and least proper values are M and m. Let ϕ be the angle made by the radius vector r at the point P of the hyperellipsoid $rAr=1$ with the "principal normal plane" at P—i.e., with the subspace spanned by the vectors Ar, $A^2r, \cdots, A^kr$. Let $T_k(x)$ be the Čebyšev polynomial, with $T_k(1)=1$. Theorem: Always $\sin\phi \leq [T_k(\delta)]^{-1}$, where $\delta=(M+m)(M-m)^{-1}$. The proof, which is expressed in terms of multivectors, is long.

The theorem is used to estimate the convergence of a proposed class of gradient methods of solving a linear system $Ax=a$: Let $x_0=0$, $a_0=a$. Given x_i and $a_i=a-Ax_i$, let $x_{i+1}-x_i$ be so chosen in the k_i-dimensional ($k_i \geq 1$) space Π_i spanned by $a_i, Aa_i, \cdots, A^{k_i-1}a_i$ that $|Ax_{i+1}-a|$ is minimized. It is shown that $|a_{i+1}| \leq |a_i| \sin\phi_i$, where ϕ_i is the angle made by a_i with the plane $A\Pi_i$.

For $k_i=1$ the author's method is the "1-process" described also by Krasnosel'skiĭ and Kreĭn [Mat. Sbornik N.S. 31(73), 315–334 (1952); these Rev. 14, 692, q. v.]. The author states that the 1-process requires one multiplication of A by a vector per step, whereas the related "0-process" of Kantorovič and others takes two. [The reviewer can find no such difference.]

The reviewer suspects the theorem could be proved briefly and elegantly following the Birman paper cited by the author [Uspehi Matem. Nauk (N.S.) 5, no. 3(37), 152–155 (1950); these Rev. 12, 32; 14, 412].

G. E. Forsythe (Los Angeles, Calif.)

KRASNOSEL'SKII, M. A.

Krein, M. G., and Krasnosel'skiĭ, M. A. Stability of the index of an unbounded operator. Mat. Sbornik N.S. 30(72), 219–224 (1952). (Russian)

Let A be a linear operator from the Banach space E_1 into the Banach space E_2 such that the set of values $R(A)$ is a closed subspace, [illegible] whose zeros $N(A)$ has finite dimensionality $n(A)$ and such that the factor-space $E_2/R(A)$ has finite dimensionality $d(A)$. The authors prove for the index $\chi(A) = n(A) - d(A)$ two results in extension of results of the reviewer [Mat. Sbornik N.S. 28(70), 3–14 (1951); these Rev. 13, 46] where the notation $\kappa(A)$ was used for $\chi(A)$. The first is that there is a positive ε such that for bounded B with $\|B\| < \varepsilon$, $A+B$ satisfies the same conditions as A, with $\chi(A+B) = \chi(A)$, $d(A+B) \leq d(A)$; the novelties here are that A need not be bounded, E_2 need not be the same as E_1, and the result for $d(A)$. The second result states that $\chi(A+B) = \chi(A)$ if B is bounded and finite-dimensional or if B is completely continuous and E_2 has a basis; in this case the extension is partial in that the reviewer did not require the existence of a basis. The authors speculate that the basis restriction is not essential. Full proofs are given, on different lines to those of the reviewer. There is close connection with the authors' joint work with D. P. Mil'man [Sbornik Trudov Inst. Mat. Akad. Nauk Ukrain. SSR 11, 97–112 (1948); not accessible to the reviewer].

F. V. Atkinson (Ibadan).

Source: Mathematical Reviews, Vol. 13 No. 6

KRASNOSEL'SKIY, M. A.

Mathematical Reviews
Vol 14 No. 7
July - August, 1953
Numerical and Graphical Methods.

Krasnosel'skiĭ, M. A., and Kreĭn, S. G. An iteration process with minimal residuals. Mat. Sbornik N.S. 31(73), 315–334 (1952). (Russian)

Let B be a real positive definite matrix. The authors introduce the "α-processes," a family of gradient methods for solving a system of linear equations $Bx=b$, depending on a real parameter α. Let x_0^α be arbitrary. For each $k=0, 1, \cdots$, a sequence $\{x_k^\alpha\}$ converging to $x^* = B^{-1}b$ is defined by letting $x_{k+1}^\alpha = x_k^\alpha - c_\alpha \Delta_k^\alpha$, where $\Delta_k^\alpha = Bx_k^\alpha - b$, and where $c_\alpha = (B^\alpha \Delta_\alpha^\alpha, \Delta_\alpha^\alpha)/(B^{\alpha+1}\Delta_\alpha^\alpha, \Delta_\alpha^\alpha)$. (In x_k^α and Δ_k^α, α is a superscript; but B^α is the αth power of B.)

For real γ, let $\|z\|_\gamma$, the "γ-length" of z, be $(z, B^\gamma z)^{\frac{1}{2}}$. Then the α-process selects x_{k+1}^α among all vectors of the form $x_k^\alpha - \gamma\Delta_k^\alpha$ $(-\infty < \gamma < \infty)$ so as to minimize $\|x_{k+1}^\alpha - x^*\|_{\alpha+1} = \|\Delta_{k+1}^\alpha\|_{\alpha-1}$. By a very simple argument it is shown that all α-processes have the same norm, i.e., one always has

$$\|\Delta^\alpha_{k+1}\|_{\alpha-1} \leq \left(\frac{M-m}{M+m}\right) \|\Delta_k^\alpha\|_{\alpha-1},$$

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(over)

where m, M are the least, greatest eigenvalues of B, and equality is attained for some x_k^*. (Details are missing on the matter of equality.) In a preliminary theorem it is shown that $\log (B^\alpha x, x)$ is a convex function of α.

The 0-process is the method of steepest descent of Kantorovich and others, while the 1-process is proposed for practical use and given the name of the title. The choice between the two depends on what measure of the error one is striving to reduce. In addition to the minimizing nature of the 0- and 1-processes inherent in the above, it is shown that the 0-process is the best of the "practically realizable" α-processes (i.e., $\alpha=0, 1, 2, \cdots$) in the one-step reduction of $\|x_k-x^*\|_2$. The non-linear transformation L: $x_k^1 \rightarrow x^1_{k+1}$ is studied in some detail; it is shown that L is commonly $n-1$ to 1. *G. E. Forsythe* (Los Angeles, Calif.).

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Operators, Hermitian 21 Jan 52

"Separation of Operators Operating From Space L^q Into Space L^p," M. A. Krasnosel'skiy, Inst of Math, Acad Sci Ukrainian SSR

"Dok Ak Nauk SSSR" Vol LXXXII, No 3, pp 333-335

Obtain such representation as $A = HH^*$ for certain operators defined in L^q, as a natural generalization of the operator representation $A = A^{1/2} \cdot A^{1/2}$. Subject problem arose in connection with one of the works of M. Golomb (Math Zs, 39, 1, 1934). Submitted by Acad A. N. Kolmogorov 23 Nov 51.

211T72

KRASNOSEL'SKIY, M.A.

USSR/Mathematics - Operators, Linear Integral 1 Jul 52

"Linear Integral Operators in Orlicz Spaces," M. A. Krasnosel'skiy, Ya. B. Rutitskiy

"Dok Ak Nauk SSSR" Vol LXXXV, No 1, pp 33-36

Studies the following linear integral operator: $Au(x) = \int_G K(x,y)u(y)dy$, where $K(x,y)$ is a function measurable on $\bar{G}$ (topological product GxG, where G is a compact set in n-dimensional Euclidean space); also explains when it will be a continuous operator operating from one Orlicz space $L_{M_1}(G)$ to another

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space $L_{M_2}(G)$. A. Zaanen has investigated this operator A (see Ann of Math, 47, No 4, 1946. Submitted by Acad A. N. Kolmogorov 28 Apr 52.

224T85

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Operator Index

Jul/Aug 53

"Index of Unbounded Operator," I. Ts. Gokhberg, Soroki, Moldavian SSR

Mat Sbor, Vol 33 (75), No 1, pp 193-198

Considers: a linear operator A that acts from a certain Banach space E_1 into a certain Banach space E_2; the region D(A) that defines A; the region R(A) of A's values. Assumes that the operators A possess the following properties: Ax=0 has a finite number of linearly independent solutions T(A) of measure a(A); the factor-space $E_2/\overline{R(A)}$ has finite measure b(A); the form of A is closed (i.e..A is normally solvable). Demonstrates two theorems. Cites the related work of M. G. Kreyn and M. A. Krasnosel'skiy ("Stability of the Index of an Unbounded Operator," Mat Sbor, Vol 30 (72), 1952). Presented 1 Oct 52.

271T88

KRASNOSEL'SKIY, M. A.

Mathematical Reviews
May 1954
Analysis

10-7-54

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Krasnosel'skiĭ, M. A. Application of variational methods to the problem of branch points. Mat. Sbornik N.S. 33(75), 199–214 (1953). (Russian)

Suppose A is a nonlinear operator in a real Hilbert space H, which leaves the zero element θ invariant. If $A\varphi_0=\lambda_0\varphi_0$, $\varphi_0\neq\theta$, then λ_0 is an eigenvalue, φ_0 an eigenvector of A. λ_0 is a branch point of A if for every positive ϵ, δ there exists an eigenvalue λ and eigenvector φ for which $|\lambda-\lambda_0|<\epsilon$, $\|\varphi\|<\delta$. The main result of this paper is contained in the following theorem. Suppose the completely continuous operator Γ $(\Gamma\theta=\theta)$ is the gradient of a weakly continuous functional Φ $(\Phi(\theta)=0)$, and Γ has at θ a Fréchet derivative B (linear) which is completely continuous and self-adjoint. Then every eigenvalue of B is a branch point of Γ. The proof of this theorem is based on a minimax construction for the functional Φ, along the lines developed by Lyusternik and Šnirelman [Uspehi Matem. Nauk (N.S.) 2, no. 1(17), 166–217 (1947); these Rev. 10, 624]. The results are applied to the nonlinear integral equation $\lambda\varphi(x)=\int_G K(x, y)f[y, \varphi(y)]dy$, $f(y, 0)\equiv 0$.

M. Golomb (Lafayette, Ind.).

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KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Integral Operator 11 Feb 53

"Certain Properties of the Root of a Linear Integral Operator," M. A. Krasnosel'skiy, Inst of Math, Acad Sci Uk SSR

DAN SSSR, Vol 88, No 5, pp 749-751

Investigates the eq of the type f = (lambda)·Gf, where operator G is the gradient of a certain functional specified in a Hilbert space and lambda is the eigenvalue. Investigates cases of more complicated operators. Presented by Acad A. N. Kolmogorov 15 Dec 52.

258T98

KRASNOSEL'SKIY, M.A.

USSR/Mathematics - Nonlinear Integral Equations 21 Feb 53

"New Theorems on Existence of Solutions of Nonlinear Integral Equations," M. A. Krasnoselskiy, Voronezh State U

DAN SSSR, Vol 88, No 6, pp 949-952

Analyzes conditions discussed by A. Hammerstein (Acta Math. 54, 1929) for solvability of Hammerstein's eqs. Considers functionals in a Hilbert space only. Presented by Acad A. N. Kolmogorov 29 Oct 52.

258T102

KRASNOSEL'SKIY, M.A.

USSR/ Mathematics - Nonlinear Integrals

1 April 1953

"Differentiability of Nonlinear Integral Operators in Orlicz Spaces", M.A. Krasnosel'skiy and Ya. B. Rutitskiy

DAN SSSR, Vol 89, No 4, pp 601-604

Investigate the operator $Hf(x)=\int_G K(x,y)\, F[y,f(y)]\, dy$, where G is a compact set of n-dimensional spaces, and show that this operator with extensive classes of kernels K(x,y) and nonlinear functions F(x,u) can be studied by means of Orlicz spaces (see A. Zygmund, Trigonometric Series, 1939). State that the general principles of functional analysis permit one to investigate the eq $f=\lambda Hf$ (H is a nonlinear operator) but finer theorems (namely, on bifurcation points, stability of solutions, eigenfunctions, etc.) are successful in establishing when H is a differential operator. One author cites earlier work (Ya. B. Rutitskiy, Dopovidi Akad Nauk RSR, No 3, 1952). Presented by Acad A.N. Kolmogorov, 2 Feb 53

256T99

KRASNOSEL'SKIY, M. A.

1 Jul 53

USSR/Mathematics - Variational Method

"Variational Methods in the Problem of Bifurcation Points," M. A. Krasnosel'skiy and A. I. Povolotskiy

DAN SSSR, Vol 91, No 1, pp 19-22

Generalize results of investigations of nonlinear operators A that operate in Banach space E and transform zero θ of this space to zero 0; namely, operators A of the form JG, where J is a certain unitary operation coinciding with unit I in one invariant subspace of linear operator B and equal to -I on the orthogonal complement, G ($G\theta=\theta$) is a gradient operator of a weakly continuous functional defined in Hilbert space H and possesses at point θ a Frechet derivative of B (this derivative a linear self-adjoint positive-definite operator). Presented by Acad A. N. Kolmogorov 22 Apr 53.

266T79

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Nonlinear Integrals

11 Sep 53

"The Structure of a Certain Operator," M. M. Vaynberg

DAN SSSR, Vol 92, No 2, pp 213-216

Considers the problem of whether a given operator h generated by a real function f(u,x) depends upon the structural properties of f(u,x), where f(u,x) is defined for all real u and for all x in the measurable set B of Euclidean space s of dimensions by the equality hu f(u(x),x). Notes that h was studied earlier by V. V. Nemytskiy (Matem Sbor. 41, 438 (1934)), by the author in 1949, and by M. A. Krasnosel'skiy (Ukrain Matem Zhurn. 2, No 3, 1951). Completes the investigation of the continuity of h for an extensive class of functional spaces, and shows that the necessary and sufficient criterion of continuity. Presented by Acad S. L. Sobolev 13 Jul 53.

269T74

KRASNOSEL'SKIY, M.A.; LADYZHENSKIY, L.A.

Conditions for total continuity of P.S.Urysohn's operator valid in the space L^p. Trudy Mosk.mat.ob-va 3:307-320 '54. (MLRA 7:7)
(Operators (Mathematics)) (Spaces, Generalized)

KRASNOSEL'SKIY, M.A.; LADYZHENSKIY, L.A.

Structure of the spectrum of positive heterogeneous operators.
Trudy Mosk.mat.ob-va 3:321-346 '54. (MLRA 7:7)
(Operators (Mathematics) (Topology)

KRASNOSEL'SKIY, M.A.

USSR/Mathematics - Nonlinear analysis

FD-1162

Card 1/1 Pub. 118-3/30

Author : Krasnosel'skiy, M. A.

Title : Some problems of nonlinear analysis

Periodical : Usp. mat. nauk, 9, No 3(61), 57-125, Jul-Sep 1954

Abstract : The author presents a survey article in which considers a number of problems in the theory of nonlinear equations, differential or integral, such as are found in mathematical physics and technology. He treats in particular the transition to operator equations (e.g. choice of space, operators of P. S. Uryson and Hammerstein, operator of Lyapunov, application of Orlicz spaces, differential operators, potential operators); the existence and uniqueness of solutions (e.g. choice of method of study, method of successive approximations, principle of the fixed point, the Lerey-Schauder method, index of solution, variational method of proving existence theorems, approximate solution by Galerkin method); eigen-functions of nonlinear operators (e.g. existence of eigen-vectors, problem of points of bifurcation, spectral study, eigenfunctions of positive operators). The author thanks V. I. Sobolev. Fifty-five references; e.g. P.S. Aleksandrov, M. M. Vaynberg, F. R. Gantmakher, M. G. Kreyn, L. A. Ladyzhenskiy, Ya. B. Rutitskiy, A. I. Povolotskiy, A. I. Nekrasov, etc.

Institution :

Submitted :

KRASNOSEL'SKIY, M.A.

2

Krasnosel'skiĭ, M. A., and Rutickiĭ, Ya. B. On a method of constructing N-functions equivalent to the complementary ones to given N-functions. Voronež. Gos. Univ. Trudy Fiz.-Mat. Sb. 33 (1954), 3–17. (Russian)

In numerous problems connected with Orlicz space L_M^* there arises the need for the set of functions containing L_N^* where M and N are complementary N-functions. For these purposes, if an M_1 can be found such that the sets L_N^* and $L_{M_1}^*$ are identical, enough is accomplished. In this paper, the authors solve the problem of finding such an M_1 for a given M from a certain class of M's.

Two N-functions M_1 and M_2 are called equivalent ($M_1 \sim M_2$) if the sets $L_{M_1}^*$ and $L_{M_2}^*$ are the same. Criteria for equivalence of M_1 and M_2 are given and these range from such an elementary one as (a) $M_1(\alpha u) \leq M_2(u) \leq M_1(\beta u)$ for some $\alpha, \beta > 0$, all u sufficiently large, to so involved a one as (b): Let M_1, M_2, N_1, N_2 be N-

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[illegible]

functions N_1 [illegible] to M_1, $M_1(u)=\int_0^u p_1(t)dt$, $N_1(v)=\int_0^v q_1(t)dt$ (these representations always exist). There is there [illegible] on the real line R such that $\mu(R\setminus T)=0$ and such that for some $k>0$

$$\lim\,[p_1(u)/q_1(u)]\ (u\in T,\ u\to\infty)=k,$$

then $M_1\sim M$, $N_1\sim N$.

By using the concept of "principal part" of an N-function M, i.e. a function Q such that $\lim M(u)/Q(u)=k>0$ as $u\to\infty$, and a class $\mathfrak{R}=\{R(u)\mid R(u)\neq 0$ for large u, $R'(u)$ exists for large u and $\lim uR'(u)/R(u)=0$ as $u\to\infty\}$, the following theorem is derived. (Note: $\mathfrak{R}$ contains all functions of the form

$$(\log u)^{\alpha_1}(\log\log u)^{\alpha_2}\cdots(\overbrace{\log\log\cdots\log}^{n} u)^{\alpha_n},$$

α_i arbitrary.)

Let M and N be complementary N-functions. Let $R(u)\in\mathfrak{R}$ and assume $M(u)R(u)$, [illegible] $N(u/R(u))R(u)$, are for large u equal to the N-functions Φ, resp. Ψ_1. Let $M(u)=\int_0^u p(t)dt$, $N(u)=\int_0^u q(t)dt$ and assume $\phi_1(u)=p(u)R(u)$ and $\phi_2(u)=q(u/R(u))$ are monotone increasing for large u and that $\lim R(q(u/R(u)))/R(u)=b>0$ as $u\to\infty$. Then $\Psi_1\sim\Psi$, where Ψ is the complementary function of Φ.

More explicit forms for linear functionals over Orlicz spaces are derived.

B. Gelbaum.

2/2

Krasnosel'skiĭ, M. A. On a proof of Helly's theorem on sets of convex bodies with common points. Voronež. Gos. Univ. Trudy Fiz.-Mat. Sb. 33 (1954), 19–20. (Russian)

A proof of Helly's theorem is based on the fact that if an n-simplex is covered by $n+1$ closed sets so that the ith set misses the ith face of the simplex, then these sets have a common point [Knaster, Kuratowski, and Mazurkiewicz, Fund. Math. 14 (1929), 132–137]. *V. L. Klee.*

KRASNOSEL'SKIY, M.A.; RUTITSKIY, Ya.B.

Linear functionals in Orlicz spces. Dokl. AN SSSR 97 no.4:581-584
Ag '54. (MLRA 7:9)

1. Predstavleno akademikom P.A.Aleksandrovym
(Functional analysis) (Spaces, Generalized)

KRASNOSEL'SKIY, M. A.

USSR/Mathematics

Card : 1/1 Pub. 22 - 6/48

Authors : Krasnosel'skiy, M. A.

Title : Splitting the linear integral operators acting from one Orlicz space into another.

Periodical : Dok. AN SSSR 97/5, 777 - 780, August 11, 1954

Abstract : A series of theorems are proved in order to legalize the reduction of the integral operator $A\varphi(s) = \int_{G} K(s,t)\varphi(t)dt$, used in analysis of non-linear equations by the method of calculus of variations, to the form $A = HH^{*}$. Eight references (1931-1953).

Institution : Voronezh State University

Presented by : Academician A. N. Kolmogorov, May 27, 1954

KRASNOSEL'SKIY, M. A.

USSR/Mathematics - Topology

Card 1/1 : Pub. 22 - 3/44

Authors : Krasnosel'skiy, M. A.

Title : On stability of critical meanings of even functionals over a sphere

Periodical : Dok. AN SSSR 97/6, 957-959, Aug 21, 1954

Abstract : Lusternak's theorem on special properties of even functionals having an infinite number of critical meanings on every sphere in Hilbert's space is analyzed and criticized. Four references: (1930-1953).

Institution : Voronezh State University

Presented by : Academician P. S. Alexandrov, May 27, 1954

KRASNOSEL'SKIY, M.A.

Two remarks on the method of sequential approximations. Usp. mat.nauk. 10 no.1:123-127 '55 (MIRA 8:6)
(Approximate computation)(Topology)

KRASNOSEL'SKIY, M. A.

Krasnosel'skiĭ, M. A., and Kreĭn, S. G. On the principle of averaging in nonlinear mechanics. Uspehi Mat. Nauk (N.S.) 10 (1955), no. 3(65), 147–152. (Russian)

1-F/W

MS

Given the system

(1) $$\dot{x}=\epsilon X(x, t)$$

with x, X n-vectors and x varying in a bounded domain D of E^n, suppose that for every x in D

(2) $$\lim_{T\to+\infty}\frac{1}{T}\int_0^T X(x, t)dt=X_0(x)$$

exists. Take now the system

(3) $$\dot{y}=\epsilon X_0(y)$$

and let $x(t)$, $y(t)$ be solutions of (1) and (3) such that $x(0)=y(0)=x_0$. Bogoliubov proved [On some statistical methods in mathematical physics, Akad. Nauk Ukrain. SSR, 1945; MR 8, 37] the following theorem. Let $X(t, x)$ be bounded in D and satisfy there a Lipschitz condition with constant independent of x, t. Let also the limit (2) exist for every x in D. Suppose finally that $y(t)$ is known for $\epsilon=1$ and $t\in[0-T]$ and together with a certain neighborhood does lie in D. Then, given $\eta>0$, there exists $\epsilon_0>0$ such that for $0<\epsilon<\epsilon_0$, $x(t)$ as defined above is in modulus within an η-neighborhood of $y(t)$ on $t\in[0-T/\epsilon]$.

(over)

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6

Krasnosel'skiy, M.A., and Krein, S.G.

It was shown by Gikhman [Ukrain. Mat. Ž. 4 (1952), 215–218 (unavailable for review)] that the above theorem is a ready consequence of a theorem on the continuous dependence of the solution of a differential equation on a parameter. Gikhman leaned heavily upon the Lipschitz condition. His result is proved here under much more general conditions, and this extends considerably the reach of the theorem of Bogoliubov. *S. Lefschetz.*

2/2

KRASNOSEL'SKIY, M.A.

USSR

Krasnosel'skiĭ, M. A., and Sobolev, V. I. Conditions of [illegible] Izv. Akad. Nauk SSSR [illegible] (1953) [illegible] (Russian)

[illegible] The establish[illegible] (G) and [illegible] (D) where [illegible] (G) [illegible] (G) [illegible] closed set in [illegible] Banach space [illegible] (2) The [illegible] satisfies the A-condition [illegible] constant. The [illegible] two previous papers by the first-named author [Krasnosel'skiĭ, Doklady Akad. Nauk SSSR (N.S.) 77, [illegible] (1951); 89, 601–604 (1953); MR 13, 357; 15, [illegible]] [illegible] of these references the relevance of the [illegible] to the study of nonlinear integral operators [illegible] Proofs in this paper [illegible]

B. R. Gelbaum

KRASNOSEL'SKIY, M.A., (Voronezh)

Stability of critical values of even functionals on a shere.
Mat.sbor.37 no.2:301-322 S-O '55. (MLRA 9:1)
(Functional analysis) (Topology)

KRASNOSEL'SKIY, M-A.

Krasnosel'skiĭ, M. A. On computation of the rotation of a vector field on the n-dimensional sphere. Dokl. Akad. Nauk SSSR (N.S.) 101 (1955), 401–404. (Russian) 1 - F/W

Let U be a periodic homeomorphism of the Euclidean n-sphere S^n onto itself with period p, so that $U^p x = x$ for $x \in S^n$. Let γ_U be the degree of the mapping U. Let V be a periodic homeomorphism of Euclidean $(n+1)$-space R^{n+1} onto itself, the degree of which is γ_V and the period of which is q, with p divisible by q and $\|Vx\| = \|x\|$ for $x \in R^{n+1}$. The author's main result is as follows: Suppose none of the mappings $U^1, \cdots, U^{p-1}$ has a fixed point. Let Φ and Ψ be continuous vector fields without null vectors on S^n, satisfying the conditions MS

$$\Phi U x = V \Phi x, \; \Psi U x = V \Psi x \quad (x \in S^n).$$

Then $\gamma_\Phi = \gamma_\Psi$ if $\gamma_U \gamma_V = -1$, and $\gamma_\Phi = \gamma_\Psi \pmod{p}$ if $\gamma_U \gamma_V = 1$. *J. M. Danskin* (Princeton, N.J.).

Voronezh State U

KRASNOSEL'SKIY, M. A.

Krasnosel'skii, M. A., and Krein, S. G. Nonlocal existence theorems and uniqueness theorems for systems of ordinary differential equations. Dokl. Akad. Nauk SSSR (N.S.) 102 (1955), 13-16. (Russian) 1-F/W

Let E denote a Banach space. For $x \in E$, $-\infty < t < \infty$, let $f(x, t) \in E$ and assume that $\|x\| < a$ and $|t| \leq a$ imply $\|f(x, t)\| \leq M(a) < \infty$. First a local existence theorem is stated for the equation $\dot{x} = f(x, t)$ subject to $x(t_0) = x_0$, with f continuous, $f = f_1 + f_2$ with f_1 completely continuous and f_2 Lipschitz-continuous. There follow two uniqueness theorems and two theorems on the continuation of solutions over the interval $t_0 \leq t < \infty$. Suppose that $L(u) \geq 0$ is continuous for $0 \leq u < \infty$ and that $\psi(t) \geq 0$ is integrable for $0 \leq t_1 \leq t \leq t_2 \leq \infty$. One existence theorem states that if there is a functional $\Psi(x)$ satisfying a uniform Lipschitz condition with $\Psi(x) \to \infty$ as $\|x\| \to \infty$, such that

$$\|f(x, t)\| \leq L(\Psi(x))\psi(t),$$

and if $\int_T^\infty du/L(u) = \infty$ then a solution can be continued for $t_0 \leq t < \infty$. The corresponding uniqueness theorem requires that $\Psi(0) = 0$, that Ψ satisfy a Lipschitz condition in a neighborhood N of the origin, that $x, y \in N$ imply $\|f(x, t) - f(y, t)\| \leq L(\Psi(x - y))\psi(t)$, and that, for $\epsilon > 0$, $\int_0^\epsilon du/L(u) = \infty$, and concludes that solutions with the same initial data are identical. The other theorems are similar but more complicated. Applications include systems in finite-dimensional space with various norms.

F. A. Ficken (Knoxville, Tenn.).

KRASNOSEL'SKIY, M.A.

Krasnosel'skii, M. A. On special coverings of a finite-dimensional sphere. Dokl. Akad. Nauk SSSR (N.S.) [illegible] (Russian)

[illegible] mapping of the n-dimensional unit [illegible] satisfying two conditions: each of the mappings [illegible] has no fixed point on S^n, and [illegible] is the identity mapping. A closed set [illegible] and it is [illegible] if there is a pair [illegible] obtained from S^n by identifying [illegible] The author [illegible]

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Integral equations CARD 1/3 PG - 166
AUTHOR BACHTIN I.A., KRASNOSELSKIJ M.A.
TITLE To the problem on the longitudinal flexure of a beam of variable flexural rigidity.
PERIODICAL Doklady Akad. Nauk 105, 621-624 (1955)
reviewed 7/1956

The author uses the method of the non-linear functional analysis for the investigation of the longitudinal flexure of a thin beam of variable flexural rigidity which is fastened by a hinge. One end of the beam can move in the horizontal plane. The corresponding differential equation be

(1) $$\frac{d^2y}{ds^2} = -P\varrho(s)y\sqrt{1-\left(\frac{dy}{ds}\right)^2}$$

with the boundary conditions

(2) $$y(0) = y(1) = 0$$

(P is the charge, $\varrho(s)$ the flexural rigidity, s the length of the curved beam, y the corresponding deviation from the equilibrium position). By $\frac{d^2y}{ds^2} = -\psi(s)$ the solution of this equation can be reduced to the determination

Doklady Akad. Nauk 105, 621-624 (1955) CARD 2/3 PG - 166

of $\varphi(s)$ of the integral equation $\varphi(s) = PB\,\varphi(s)$, where

$$B\varphi(s) = \varrho(s)\int_0^1 G(s,t)\,\varphi(t)\,dt\left(1-\left[\int_0^1 G_s'(s,t)\,\varphi(t)\,dt\right]^2\right)^{-\frac{1}{2}},$$

and the determination of $y(s)$ of

$$y(s) = A\varphi(s) = \int_0^1 G(s,t)\,\varphi(t)\,dt$$

$$G(s,t) = \begin{cases} s(1-t) & \text{for } s \leq t \\ t(1-s) & \text{for } t \leq s. \end{cases}$$

The operator B is considered on the sphere $T \subset C$ (C the space of the functions being continuous on $[0,1]$) of radius 1/2. It is complete on T and differentiable according to Frechet, where its Frechet's derivative in the zero point of the space is the operator $D\varphi(s) = \varrho(s)A\,\varphi(s)$. If $0 \leq \varphi(s) \leq 1/2$, then $B[t\,\varphi(s)] \geq tB\,\varphi(s)$ $(0 \leq t \leq 1)$. If $\varphi_1(s) \geq \varphi_2(s)$ $(0 \leq \varphi_1(s), \varphi_2(s) \leq 1/2$, $\varphi_1(s) \not\equiv \varphi_2(s))$, then there exists an α such that $B\,\varphi_1(s) - B\,\varphi_2(s) \geq \alpha\varrho(s)s(1-s)$. The charge P_0 is called critical if for arbitrary $\varepsilon, \delta > 0$ there exists a

Doklady Akad. Nauk 105, 621-624 (1955) CARD 3/3 PG - 166

solution of (1)-(2) being different from zero, which satisfies the inequation $|y(s)| < \varepsilon$ if at the same time $|P-P_o| < \delta$. The critical forces of the considered problem agree with the eigenvalues P_k of the boundary value problem

$$\frac{d^2y}{ds^2} = P\,\xi(s)y \qquad y(0) = y(1) = 0.$$

The investigation of the question, when (1)-(2) admits small solutions, yields the theorem: For critical charges P_k $(k=1,2,\ldots)$ the equation $\varphi(s) = PB\,\varphi(s)$ has no small solutions being different from zero. To every P_k there corresponds an interval $\Delta_k = (P_k, P_k + h_k^2)$ such that for $P \in \Delta_k$ the equation $\varphi(s) = PB\,\varphi(s)$ has solutions being different from zero, which for $P \rightarrow P_k$ tend to zero together with their second derivatives. The proofs of the theorems and lemmas are sketched.

INSTITUTION: Public University Voronež.

KRASNOSEL'SKIY, M.A.

Transactions of the Third All-union Mathematical Congress (Cont.) Moscow, Jun-Jul '56, Trudy '56, V. 1, Sect. Rpts., Izdatel'stvo AN SSSR, Moscow, 1956, 237 pp.

Krasnosel'skiy, M. A. (Voronezh). On the Investigation of Bifurcation Points of Non-linear Equation. 204-205

Kreyn, S. G. (Voronezh). Mathematical Problems in the Theory of Motion of Solid Bodies With Fluid-filled Cavities. 205

Kupradze, V. D. (Tbilisi). On Some New Research at the University of Tbilisi in the Mathematical Theory of Elasticity. 205

Mikhaylov, G. K. (Moscow). Precise Solution of a Problem on Stabilized Motion of Ground Water in Vertical Plane With Free Surface and Feeding Zone. 205-206

Mention is made of Polubarinova-Kochina, P. Ya.

Movchan, A. A. (Moscow). Linear Oscillations of a Plate Moving in Gas at High Velocity. 206

Card 68/80

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/1 PG - 544
AUTHOR KRASNOSEL'SKIJ M.A.
TITLE Topological methods in the theory of non-linear integral equations. (Modern problems of mathematics).
PERIODICAL Moscow: State Publication for technical-theoretical literature 392 p. (1956)
reviewed 1/1957

In the present book the author compiles most of the researches on the non-linear analysis in the Banach spaces, researches combined essentially with the method of Leray-Schauder. The book contains six chapters. In the first chapter the author studies the integral operators to which the abstract methods in the following chapters are applied. The second chapter contains the notions and the fundamental theorems: The rotation of vector fields (in the sense of the author being equivalent to the topological degree of Leray-Schauder), the theorems of Brouwer, Hopf, Leray-Schauder, Kusternik-Šnirel'-man-Borsuk.
The notions and theorems of the combinatoric topology used in this theory are deduced partially. Then (Chapter III and IV) these methods are applied to more concrete problems: the existence of solutions and proper values, ramification points, non-linear spectral analysis (the author defines a resolvent for non-linear operators), asymptotically linear operators, Liapunov theorems.

SOV/124-57-4-3911

Translation from: Referativnyy zhurnal. Mekhanika, 1957, Nr 4, p 11 (USSR)

AUTHOR: Krasnosel'skiy, M. A.

TITLE: On the Investigation of Points of Forking of Nonlinear Equations (Ob issledovanii tochek bifurkatsii nelineynykh uravneniy)

PERIODICAL: Tr. 3-go Vses. matem. s"yezda. Vol I. Moscow, AN SSSR, 1956, pp 204-205

ABSTRACT: Bibliographic entry

Card 1/1

APPROVED FOR RELEASE: Monday, July 31, 2000 CIA-RDP86-00513R000826120(

Krasnoselskiy, M.A.

Krasnosel'skiĭ, M. A.; and Rutickiĭ, Ya. B. General theory of Orlicz spaces. Voronež. Gos. Univ. Trudy Sem. Funkcional. Anal. no. 1 (1956), 3-38. (Russian)

A general and thorough exposition of the theory of Orlicz spaces is presented. The main theorems of the subject are stated and proved and many interesting illustrations are provided. Although in its service as a synthesis the paper merits attention, its value to the experienced analyst in the field is limited since few new results (save for elegant reformulations) are given.

B. Gelbaum (Minneapolis, Minn.)

1-FW

4

KRASNOSEL'SKIY, M.A.

Krasnosel'skii, M. A., and Krein, S. G. On the theory of ordinary differential equations in Banach spaces. Voronež. Gos. Univ. Trudy Sem. Funkcional. Anal. no. 2 (1956), 3–23. (Russian) 1-FW

3

Slight extensions and detailed proofs of results stated by the same authors in Dokl. Akad. Nauk SSSR (N.S.) 102 (1955), 13–16 [MR 17, 151]. *F. A. Ficken.* 1/1

KRASNOSEL'SKAYA, [illegible]

[illegible] conditions [illegible] and the pair [illegible] is continuous [illegible] Details are extremely [illegible]

[illegible] (Minneapolis, Minn.)

KRASNOSEL'SKIY, M. [illegible]

[illegible]

APPROVED FOR RELEASE: Monday, July 31, 2000 CIA-RDP86-00513R00082612O(

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Theory of approximations CARD 1/1 PG - 429
AUTHOR KRASNOSEL'SKIJ M.A.
TITLE On some approximative methods for the determination of the eigenvalues and eigenvectors of a positive definite matrix.
PERIODICAL Uspechi mat. Nauk 11, 3, 151-158 (1956) reviewed 12/1956

The author proposes some methods for the approximative computation of the eigenvalues and eigenvectors of a positive definite, quadratic, symmetric matrix of n-th order. The matrix is considered as an operator in the E^n. The proposed methods are analytic analogues to the well-known methods for the construction of point sequences x_k $(k=0,1,2,\ldots)$ on the ellipsoid $(Ax,x) = 1$ which converge to the endpoint of one of the semiaxes. Compare Kantorovič (Uspechi mat. Nauk 3, 6, 89-185 (1948)), Kostarčuk (Doklady Akad. Nauk 98, 531-534 (1954)), Lanczos (Journ. of Research of the Nat.Bureau of Standards 45, 200 (1950)) etc..

KRASNOSEL'SKIY, M.A.; SOBOLEV, V.I.

The Voronezh Seminar on functional analysis. Usp.mat.nauk 11 no.5: 249-250 S-O '56. (MLRA 10:2)
(Voronezh--Functional analysis)

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/1 PG - 410
AUTHOR KRASNOSEL'SKIJ M.A.
TITLE On a boundary value problem.
PERIODICAL Izvestija Akad. Nauk 20, 241-252 (1956)
reviewed 12/1956

For the non-linear boundary value problem

$$y'' = f(x,y,y')$$
$$y(0) = y(\pi) = 0$$

the author gives new conditions for the existence of the solution. If the boundary value problem possesses a trivial solution, the author gives conditions for the existence of a second, non-vanishing solution. Methods of the functional analysis are used.

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Integral equations CARD 1/3 PG - 368
AUTHOR KRASNOSEL'SKIJ M.A.
TITLE On the equations of A.I.Nekrasov of the theory of surface waves of a heavy liquid.
PERIODICAL Doklady Akad. Nauk 109, 456-459 (1956)
reviewed 11/1956

Nekrasov has shown that the non-vanishing solutions of the integral equation

$$\varphi(x) = \mu \int_0^{2\pi} \frac{K(x,y)\sin\varphi(y)}{1 + \mu\int_0^y \sin\varphi(t)\,dt}\,dy$$

(1)

$$K(x,y) = \sum_{n=1}^{\infty} \frac{\sin nx \sin ny}{\mu_n}$$

determine the form of the waves on the surface of a heavy liquid. Here the positive eigenvalues μ_n are different in dependence of the fact if the depth is finite or infinite. The papameter μ is determined by the characteristics of the stream. The author applies methods of the functional analysis and topological considerations in order to investigate the solutions of this integral equation without constructing the solution. The initial point of the investigation is the statement that the operator

Doklady Akad. Nauk 109, 456-459 (1956) CARD 2/3 PG - 368

$$A(\varphi,\mu) = \mu \int_0^{2\pi} \frac{K(x,y)\sin\varphi(y)}{1+\mu\int_0^y \sin\varphi(t)\,dt}\,dy$$

is completely continuous on a sufficiently small sphere of functions being continuous on $[0,2\pi]$, and admits the representation

$$A(\varphi,\mu) = \mu B\varphi + C(\varphi,\mu) + D(\varphi,\mu).$$

Here B is a linear integral operator which is determined by the kernel $K(x,y)$,

$$C(\varphi,\mu) = -\mu^2 \int_0^{2\pi} K(x,y)\,\varphi(y)\left[\int_0^y \varphi(t)\,dt\right] dy$$

and $D(\varphi,\mu)$ is of higher order than $C(\varphi,\mu)$ in φ. Now from an earlier result of the author follows that (1) possesses small non-vanishing solutions for certain μ, which lie in the neighborhood of each μ_n. In order to find these μ-values the author applies very interesting topological considerations (see: Bachtin and Krasnosels'kij, Doklady Akad. Nauk 105, 4, (1955)) which

Doklady Akad. Nauk 109, 456-459 (1956) CARD 3/3 PG

lead to the theorems which reduce the computation to a minimum at the applications. The capable application of these methods enables the author to make very useful assertions on the distribution, existence, uniqueness and number of the non-vanishing solutions of Nekrasov's equations and some similar ones.

INSTITUTION: University, Voronež.

KRASNOSEL'SKIY, M.A.

CARD 1/3 PG - 711

SUBJECT USSR/MATHEMATICS/Functional analysis
AUTHOR KRASNOSEL'SKIJ M.A., KREJN S.G., SOBOLEVSKI P.E.
TITLE On differential equations with bounded operators in Banach spaces.
PERIODICAL Doklady Akad.Nauk 111, 19-22 (1956)
reviewed 4/1957

The authors consider the equation

(1) $$\frac{dx}{dt} = A(t)x + f(t,x),$$

where $x(t)$ is the sought function with a range of values in the Banach space E, $A(t)$ and $f(t,x)$ are operators in E and besides $A(t)$ is unbounded, closed and linear for every t. A solution is sought which satisfies the initial condition

(2) $$x(0) = x_0,$$

where x_0 belongs to the region of definition $D(A)$ of the operator $A(0)$. The authors use the theory of semigroups and therefore it is assumed that $A(t)$ is the generating operator of a strongly continuous semigroup of bounded operators $T(\xi)$ $(\xi > 0)$ for every t. At first the linear equation

$$\frac{dx}{dt} = Ax + f(t)$$

is considered, where A is independent of t. Let Q be the linear operator

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Doklady Akad.Nauk 111, 19-22 (1956) CARD 2/3 PG - 711

$$Qx(t) = \int_0^t T(t-x)x(\tau)d\tau.$$

Theorem: a) Q acts and is continuous in the space C_L of functions which satisfy the Lipschitz condition. If for $\xi > 0$ the semigroup $T(\xi)$ is continuous with respect to the norm (condition C_n according to Hill), then Q acts from C_L to C_1 and is continuous. b) if A^{-1} is completely continuous, then Q as an operator from C_L to C is completely continuous too.

Theorem: Let $T(\xi)$ satisfy the condition C_n and let f(t) be continuous and have a strongly bounded variation. For $x_0 \in D(A)$ the formula

$$x(t) = T(t)x_0 + Qf(t)$$

yields the solution of (1)-(2).

Let be given a homogeneous linear equation $\frac{dx}{dt} = A(t)x$ and let be satisfied the condition α) $C(t) = A(t)\frac{d}{dt}A^{-1}(t)$ bounded and strongly continuous in t.

Theorem: If α) is satisfied, then 1) the operators A(t) have a common region

of definition, 2) the operators $B(t,s) = A(t)A^{-1}(s)$ are continuous with respect to the norm in t and s and 3) the derivative $\frac{\partial B(t,s)}{\partial t}$ is strongly continuous for every s in t.
If 1) and 3) are satisfied, then α) is satisfied too. This theorem and a further one are incdirect connection with the investigations of Kato (J.Math.Soc.Jap. 5, no.2, (1953)).
Then the non-linear equation (1) is treated. A generalized solution of (1)-(2) means a function x(t) which satisfies the operator equation

$$(3) \qquad x(t) = Qf\left[t,x(t)\right] + U(t,0)x_0 \, .$$

For the proof of the theorems of existence theorems of fixed points are used. For a sufficient smoothness of f(t,x) in some cases it can be shown that the generalized solutions the existence of which was proved, are ordinary solutions of (1). Some examples are considered.

~~KRASNOSEL'SKIY, M.A.~~ KRASNOSEL'SKIY M.A.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/2 PG - 612
AUTHOR KRASNOSEL'SKI M.A.
TITLE On the application of the methods of non-linear functional analysis to some problems on periodic solutions of equations of non-linear mechanics.
PERIODICAL Doklady Akad.Nauk 111, 283-286 (1956)
reviewed 2/1957

With means of non-linear functional analysis the following questions of non-linear mechanics are treated: Existence of periodic solutions, uniqueness of them, dependence of these solutions on the parameters of the right side of an equation etc. The starting point of the considerations is the statement that to every system of ordinary differential equations there can be associated an equation with a completely continuous operator such that the solutions of this equation only determine the periodic solutions of the system. By aid of theorems on fixed points the author obtains sufficient conditions for the existence of periodic solutions. E.g. let be given the system

$$(1) \qquad \ddot{x}_i + g_i(t,x_1,\ldots,x_n,\dot{x}_1,\ldots,\dot{x}_n) = 0 \qquad (i=1,\ldots,n),$$

where the g_i are continuous and possess the period 2π in t. If the condition

$$(2) \qquad \sum_{i=o}^{n} x_i g_i(t,x_1,\ldots x_n;y_1,\ldots y_n) \leq a \sum_{i=1}^{n} x_i^2 + b \sum_{i=1}^{n} |y_i|^{2-\gamma} + c$$

Doklady Akad.Nauk 111, 283-286 (1956) CARD 2/2 PG - 612

with $0 < \gamma < 2$; a,b,c - numbers and $a < 0$ is satisfied, then (1) has at least one periodic solution. If besides the g_i satisfy the condition

$$g_i(-t,-x_1,\dots -x_n,y_1,\dots y_n) = -g_i(t,x_1,\dots,x_n,y_1,\dots y_n)$$

or

$$g_i(t+\pi,-x_1,\dots,-x_n,-y_1,\dots -y_n) = -g_i(t,x_1,\dots,x_n,y_1,\dots,y_n),$$

then for the existence of a periodic solution the condition (2) with $a < 1$ is sufficient.
Further theorems relate to systems

$$\ddot{x}_i+g_i(x_1,\dots,x_n;\ \dot{x}_1,\dots,\dot{x}_n) = 0 \quad \text{and} \quad \ddot{x}_i+g_i(x_1,\dots,x_n) = 0.$$

INSTITUTION: University, Voronež.

SUBJECT USSR/MATHEMATICS/Topology CARD 1/2 PG - 752
AUTHOR KRASNOSEL'SKI M.A.
TITLE On a possible generalization of the method of orthogonal trajectories.
PERIODICAL Uspechi mat.Nauk. 12, 1, 160-162 (1957)
reviewed 5/1957

According to the topological method of Ljusternik-Šnirel'man the number of critical points of the functionals $\Phi(x)$ being defined on the space R can be estimated by constructing certain continuous deformations $\varkappa(x,t)$ which have the property that for increasing t the expressions $\Phi[\varkappa(x,t)]$ in all non-critical points x increase too. The deformations $\varkappa$ are determined by motions along the integral curves of

$$\frac{dx}{dt} = \operatorname{grad} \Phi(x).$$

These are orthogonal to the equipotential surfaces of $\Phi(x)$. The author proposes a further development of this method of orthogonal trajectories. Therefore he generalizes the notion of the continuous deformation: a family of closed connected sets $\{F(x_0,t)\}$ $(0 \leq t \leq 1)$ is called a generalized continuous deformation of the point $x_0 \in R$ if $F(x_0,0)$ consists of one point x_0 and if

Uspechi mat.Nauk 12, 1, 160-162 (1957) CARD 2/2 PG - 752

$$\lim_{t_1 \to t_2} d\left[F(x_o,t_1);F(x_o,t_2)\right] = 0,$$

where d(A,B) is the Hausdorff distance between the sets A and B. The family $\{F(x,t)\}$ of deformations of the points $x \in G$ forms a generalized continuous deformation of the set $G \subset R$ if for every connected set $G_1 \subset G$ the sets $\bigcup_{x \in G_1} F(x,t)$ are connected for all t and if

$$\lim_{t_1 \to t_2} d\left[\bigcup_{x \in G_1} F(x,t_1);\ \bigcup_{x \in G_1} F(x,t_2)\right] = 0.$$

Starting from this definition the author proposes a systematic development of the generalized method of the orthogonal trajectories. Several questions are not answered; concrete results are not given.

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/2 PG - 806
AUTHOR KRASNOSEL'SKIJ M.A.
TITLE Investigation of the spectrum of a non-linear operator in the neighborhood of the bifurcation point and application to the problem on the longitudinal bend of a compressed bar.
PERIODICAL Uspechi mat.Nauk 12, 1, 203-208 (1957) reviewed 6/1957

By the example of a compressed bar the author demonstates the application of topological methods in the bifurcation theory of small solutions. The contents of the present note in essential is contained in the author's book "Topological methods in the theory of non-linear integral equations"(1956). Interesting questions are given and partially answered. Let A be a non-linear completely continuous operator in the Banach space E, let $A\theta = \theta$. Then

(1) $$\varphi = \lambda A \varphi$$

has a solution θ for all λ. The number $\lambda_0 \neq 0$ is called bifurcation point of A if to every $\varepsilon > 0$ there corresponds a λ such that $|\lambda - \lambda_0| < \varepsilon$, to the there corresponds a solution of (1) being different from θ and if $\|\varphi\| < \varepsilon$. Let A be sufficiently smooth and permit the representation $A = B+C+D$, where B is a linear operator,

Uspechi mat.Nauk 12, 1, 203-208 (1957) CARD 2/2 PG - 806

$$C(\alpha\varphi) = \alpha^k C\varphi \; ; \; \| C\varphi_1 - C\varphi_2 \| \leq M(\|\varphi_1\| + \|\varphi_2\|)^{k-1} \; \|\varphi_1 - \varphi_2\|$$

and

$$\lim_{\|\varphi\| \to 0} \| D\varphi \| \cdot \| \varphi \|^{-k} = 0.$$

For what maximal class of operators A the set of bifurcation points of A is identical with the set of characteristic values of B ? Two partial results are given: 1) If A is the gradient of a weakly continuous functional, then the sets are identical; 2) Every characteristic value λ_0 of B with an odd multiplicity is a bifurcation point of A.
Furthermore the question is treated for which λ the equation (1) has small solutions $\neq 0$.

KRASNOSEL'SKIY, M.A.; KREYN, S.G.; MYSHKIS, A.D.

The broadened sessions of the Voronezh Seminar on Functional Analysis in March 1957. Usp.mat.nauk 12 no.4:241-250 Jl-Ag '57. (MIRA 10:10)

(Voronezh--Functional analysis)

KRASNOSEL'SKIY, M.A.; SOBOLEV, V.I.

The decomposition of linear operators. Usp.mat.nauk 12 no.4:313-317 Jl-Ag '57. (MIRA 10:10)

(Operators (Mathematics))

KRASNOSEL'SKIY, M.A.

SUBJECT USSR/MATHEMATICS/Functional analysis CARD 1/3 PG - 874
AUTHOR KRASNOSEL'SKIJ M.A., KREJN S.G., SOBOLEVSKIJ P.E.
TITLE On differential equations with unbounded operators in the Hilbert space.
PERIODICAL Doklady Akad.Nauk 112, 990-993 (1957)
reviewed 6/1957

Joining a paper of Kato (J.Math.Soc.Japan, 5, 2, (1953)) the authors investigate the equation

(1) $$\frac{dx}{dt} + A(t)x = f(t)$$

in the Hilbert space H. Kato constructed the solution of (1) in the Banach space in the form

(2) $$x(t) = U(t,0)x_0 + Qf(t),$$

where the solution of the homogeneous equation has the form

$$x(t) = U(t,s)x_0$$

with a continuous and bounded operator $U(t,s)$ and with the initial condition

$x(s) = x_0$ and

$$Qf(t) = \int_0^t U(t,s)f(s)ds.$$

In the special case considered by the authors, about U and Q more exact assertions can be made. Here it is assumed that 1) A(t) is selfadjoint and $(A(t)x,x) \geqslant (x,x)$, 2) for $0 \leq \alpha \leq 1$, $A^{-\alpha}(t)$ is differentiable, where $C_\alpha(t) = A^\alpha(t) \frac{d}{dt} A^{-\alpha}(t)$ are uniformly bounded with respect to α and t. 3) $C_1(t)$ is strongly continuous in t and bounded. It is shown that under certain conditions of 1) and 3) there follows the condition 2). Furthermore: $x(t) = U(t,s)x_0$ satisfies the homogeneous equation for all $x_0 \in H$. For $t > s$ and $0 \leq \alpha < 2$ the operators $A^\alpha(t)U(t,s)$ are bounded, where $\| A^\alpha(t)U(t,s)\| \leqslant M(t-s)^{-\alpha}$. This estimation also holds for $\alpha = 2$ if $\|C(t)-C(s)\| \leqslant L|t-s|^\beta$. The estimation holds for all α if A is constant. If f(t) satisfies the condition Lip ε with $\varepsilon \leqslant 1$, then (2) is a solution of (1) for all $x_0 \in H$ and $t > 0$. If x_0 lies in the region of definition of A, then this solution has the property $\left\| A^{\alpha}(t) \frac{dx}{dt} \right\| \leq M|t|^{-\alpha}$ for $\alpha < \varepsilon$. Let C be the space of the functions $f(t)$ being continuous on $[0,b]$ with the values in H and the norm $\|f\|_C = \max \|f(t)\|$ and let C_0' be the space of continuously differentiable functions which vanish for $t = 0$ and the norm of which is $\|f\|_{C_0'} = \max \|f'(t)\|$. If $f(t) \in C$, then it holds

$$\|Qf(t+\Delta t) - Qf(t)\| \leq K_1 \Delta t \, |\ln \Delta t| \cdot \|f\|_C ,$$

if $f(t) \in C_0'$, then we have

$$\left\| \frac{d}{dt} Qf(t+\Delta t) - \frac{d}{dt} Qf(t) \right\| \leq K_2 \Delta t \, |\ln \Delta t| \cdot \|f\|_{C_0'} .$$

If $A^{-1}(t)$ is completely continuous, then Q is completely continuous in C and C_0'. Furthermore the equation (3) $\frac{dx}{dt} + A(t)x = f(t,x)$ is considered. It is stated that the integral equation (4) $x(t) = U(t,0)x_0 + Qf[t,x(t)]$ has a solution on a certain interval. If $\|f(t+\Delta t, x+\Delta x) - f(t,x)\| \leq K(|\Delta t|^{\alpha} + \|\Delta x\|^{\alpha})$ $(\alpha \leq 1)$, then every continuous solution of (4) is also a solution of (3) for $t > 0$.

KRASNOSEL'SKIY, M.A.

20-2-5/50

AUTHOR: KRASNOSEL'SKIY, M.A.

TITLE: On Periodic Solutions in the Neighborhood of the Singular Point of a Dynamic System (O periodicheskikh resheniyakh v okrestnosti osoboy tochki dinamicheskoy sistemy)

PERIODICAL: Doklady Akademii Nauk SSSR, 1957, Vol 117, Nr 2, pp 180-183 (USSR)

ABSTRACT: Under the supposition

(1) $g_i(-x_1,\ldots, -x_n, y_1,\ldots, y_n) = - g_i(x_1,\ldots, x_n, y_1,\ldots, y_n)$

the author considers the systems

(2) $\ddot{x}_i + g_i(x_1,\ldots, x_n, \dot{x}_1,\ldots,\dot{x}_n) = 0 \quad (i = 1,\ldots,n)$.

He investigates periodic solutions with small amplitudes and the dependence of the amplitudes on the period. The obtained results are based on the author's non-linear functional-analytical investigations and on his theorems on the bifurcation points [Ref.1,3,4]. On the whole there are formulated seven theorems without proof, e.g.:

Let C denote a matrix of order n with the elements

$$c_{ij} = \frac{\partial}{\partial x_j} g_i(0,\ldots,0,0,\ldots,0) \qquad (i,j = 1,\ldots,n) .$$

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On Periodic Solutions in the Neighborhood of the Singular Point of a Dynamic System 20-2-5/50

Definition: The set $\mathfrak{N}$ of periodic solutions of (2) is said to have limit-period T, if the periods of solutions in $\mathfrak{N}$ tend to T in the case that their amplitudes go to zero. Theorem: Let $\frac{4\pi^2}{T_k^2}$ be a positive root of the character-equation of C and have an odd multiplicity. The sum of the multiplicities of the other positive roots which possess the property that $\frac{4\pi^2}{T_k^2}$ is an integral multiple of them is assumed to be even. Then an infinite set $\mathfrak{N}_k$ of periodic solutions of (2) with the limit period T_k corresponds to the root $\frac{4\pi^2}{T_k^2}$.

Theorem: Let the system $\ddot{x}_i + g_i(x_1,\dots,x_n) = 0$ $(i = 1,\dots,n)$, $g_i(-x_1,\dots,-x_n) = -g_i(x_1,\dots,x_n)$, $g_i(x_1,\dots,x_n) = \frac{\partial}{\partial x_i} G(x_1,\dots,x_n)$ $(i = 1,\dots,n)$, be given. To each positive root $\frac{2\pi^2}{T_k^2}$ of the characteristic equation of C there corresponds a set $\mathfrak{N}_k$ of periodic solutions of (2) with the limit-period T_k.

Card 2/3

On Periodic Solutions in the Neighborhood of the Singular Point of a Dynamic System 20-2-5/50

5 Soviet and 1 foreign references are quoted.

ASSOCIATION: Voronezh State University (Voronezhskiy gosudarstvennyy universitet)

PRESENTED: By N.N. Bogolyubov, Academician, 30 May, 1957

SUBMITTED: 21 May, 1957

AVAILABLE: Library of Congress

Card 3/3

AUTHOR: KRASNOSEL'SKIY, M.A., RUTITSKIY, Ya.B. 20-3-2/52

TITLE: On Some Nonlinear Operators in the Orlicz Spaces (O nekotorykh nelineynykh operatorakh v prostranstvakh Orlicha)

PERIODICAL: Doklady Akademii Nauk SSSR, 1957, Vol. 117, Nr. 3, pp. 363-366 (USSR)

ABSTRACT: Let the function $f(x,n)$ $(x \in G, -\infty < u < \infty)$ satisfy the conditions of Caratheodory. Let the operator F be defined by $Fu(x) = f[x,u(x)]$. The authors give conditions under which in a sphere of the Orlicz-space $L^*_M(G)$ this operator is differentiable according to Frechet. Further the operator $K\varphi(x) = \int_G K[x,y,\varphi(y)]\,dy$ is considered. It is shown that under certain conditions it is completely continuous; here the operator may possess also essential non-potential nonlinearities. Conditions are given under which there exists an Orlicz-space in which K is defined and completely continuous. Further the question of the differentiability of the norm in Orlicz-spaces is considered. The results can be extended also to the modulated spaces considered by Nikano. The paper contains six long theorems

Card 1/2

without proofs.
One Soviet and 2 foreign references are quoted.

ASSOCIATION: Voronezh State University (Voronezhskiy gosudarstvennyy universitet)
PRESENTED: By P.S.Aleksandrov, Academician, 30 May 1957
SUBMITTED: 30 May 1957
AVAILABLE: Library of Congress

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16(1) PHASE I BOOK EXPLOITATION SOV/1455

Krasnosel'skiy, Mark Aleksandrovich, and Yakov Bronislavovich Rutitskiy

Vypuklyye funktsii i prostranstva Orlicha (Convex Functions and Orlicz Spaces) Moscow, Fizmatgiz, 1958. 271 p. (Series: Sovremennyye problemy matematiki) 5,000 copies printed.

Ed.: M.M. Goryachaya; Tech. Ed.: V.N. Kryuchkova.

PURPOSE: This book is intended for mathematicians, senior students, aspirants, and scientific workers concerned with functional analysis and its applications, and also with various problems of the theory of functions.

COVERAGE: This book is one of a series entitled Sovremennyye problemy matematiki (Modern Mathematical Problems), published under the supervision of the editorial staff of the journal Uspekhi matematicheskikh nauk. The book presents the theory of many classes of convex functions and its applications. The material for this theory is taken from various mathematical papers. The general theory of Orlicz spaces is developed, and its applications to the

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study of operators, functionals and nonlinear integral equations are presented. The authors thank G.Ye. Shilov for his assistance in preparing the book. There are 104 references, 72 of which are Soviet, 16 English, 11 German, 3 French, and 2 Italian.

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KRASNOSEL'SKIY, M.A.

PHASE I BOOK EXPLOITATION 1087

Moskovskoye matematicheskoye obshchestvo

Trudy, t. 7 (Transactions of the Moscow Mathematical Society, v. 7) Moscow, Fizmatgiz, 1958. 438 p. 1,500 copies printed.

Editorial Staff: Aleksandrov, P.S.; Gel'fand, I.M. and Golovin, O.N.; Ed.: Lapko, A.F.; Tech. Ed.: Yermakova, Ye.A.

PURPOSE: This book presents original articles submitted to the Moscow Mathematical Society and is intended for specialists in various fields of mathematics.

COVERAGE: Volume 7 contains 12 articles concerning problems in different fields of mathematics, including functional analysis, differential geometry and mathematical logic. All contributions in this volume are Soviet. Most of the articles deal with problems of functional analysis which reflect the present-day status and trend of this branch of mathematics.

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TABLE OF CONTENTS:

The basic results given in this article were presented at the November 9, 1959 session of the Moscow Mathematical Society. The article contains the following sections:

Introduction:
1.) Certain results concerning hyperbolic equations; 2) Proof of the Uniqueness Theorem; 3) Statement of an inverse problem connected with the scattering of waves; References

The basic results given in this article were presented at the March 2, 1954 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; 1) Basic definitions; 2) Splitting of linear

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integral operators; 3) Operator f; 4) Hammerstein operator; 5) Operator G; 6) Differentiability of the Hammerstein operator; 7) Applications to theorems of the existence of solutions and to eigenfunctions; References.

Kornblyum, B.I. (Kiyev). Generalization of Wiener's Tauberian Theorem and Harmonic Analysis of Fast Increasing Functions 121

The basic results given in this article were presented at the April 23, 1954 session of the Moscow Mathematical Society. The article contains the following sections: 1) Introduction; 2) Theorem of Wiener type; 3) Lemmas on spaces $L(-\infty,\infty;\alpha)$ and $M(-\infty,\infty;\alpha)$; 4) Lemmas on Fourier transformations; 5) Lemmas on functions analytic in a strip; 6) Proof of theorem I; 7) Ideals I_γ^+ and I_γ^-; 8) General Tauberian Theorems; 9) Theorem of Berling type; 10) Spectrum of fast increasing functions; References.

Ladyzhenskaya, O.A. (Leningrad). Solution of the First Boundary Value Problem on the Large for Quasilinear Parabolic Equations 149

The basic results given in this article were presented at the December 18, 1956 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; Ch. I. A Priori Evaluations for the

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Solutions of Problems (1) and (2); 1) Evaluation of the modulus of a solution; 2) Evaluation of first derivatives of $u(x,t)$ with respect to x_k in a closed region Ω ; 3) Evaluation in the form of integrals of u derivatives contained in the equation; 4) Evaluation of the second order derivatives of u with respect to x_k in the interior of a region Ω; 5) Evaluation of the third order derivatives of u with respect to x_k; 6) Evaluation of derivatives $D^2_{tx} u$, $D^4_x u$ and $D^3_{tx^2} u$;

Ch. II. Theorems on Existence and Uniqueness of a Generalized Solution of the Boundary Value Problem; 1) Construction of Approximate Solutions; 2) Evaluation of $|\operatorname{grad} u_h (x,tp)|$; 3) Evaluation of $D^2_x u_h$ and $u_{\bar{h}t}$ in the form of integrals; 4) Proof of the existence and uniqueness theorem of a generalized solution; Ch. III. Investigation of Differential Properties of a Generalized Solution. The Existence of a Classical Solution; References.

Ryzhkov, V.V. Conjugate Systems on Multidimensional Spaces 179
The basic results given in this article were presented at the March 20, 1956 session of the Moscow Mathematical Society. This article contains

the following sections: Introduction; Ch.I. Conjugate Systems; 1) Designations and basic definitions; 2) Differential equation defining conjugate systems; 3) Condition for complete stratification of a conjugate system; Ch. II. Completely Stratifiable Conjugate Systems; 4) n-conjugate systems; 5) Conjugate Systems with one multidimensional component; 6) Completely stratifiable conjugate systems with several multidimensional components; 7) General remarks on complete stratifiable conjugate systems; References.

Fage, M.K. (Chernovitsy). Operationally Analytic Functions of One Independent Variable [Functions Defined by an Ordinary Linear Differential Operator L of an Arbitrary Order With Continuous Coefficients] 227
The basic results given in this article were presented at the October 30, 1956 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; 1) L-bases; 2) L-analytic polynomials; 3) Taylor's L-formula; 4) Taylor's L-series; 5) L-holomorphic functions; 6) L-analytic functions. Uniqueness theorem; 7) Regularly convergent sequences of L-analytic functions; 8) Operator with analytic coefficients; 9) Local equivalency of operators of an equal order; 10) Cauchy problem in the region of double operationally holomorphic functions; References.

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Levitan, B.M. Differentiation of Eigenfunction Expansion of the Schrödinger Equation 269

The basic results given in this article were presented at the October 4, 1955 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; 1) Solution of Cauchy problem; 2) Evaluation for arbitrary eigenfunctions; 3) Evaluation of derivatives of eigenfunctions in the case of an infinite region; 4) Differentiation of eigenfunction expansion; 5) The case of $q(x) \to +\infty$ at $|x| \to \infty$; References.

Men'shov, D.Ye. Limit Functions of a Trigonometric Series 291

The basic results given in this article were presented at the April 16, 1957 session of the Moscow Mathematical Society. The article contains the following sections: 1) Introduction. [Basic definitions and formulation of three theorems]; 2) [Preliminary remarks, definitions and auxiliary theorems needed to prove theorem II. Proof of theorem II[; 3) [Definitions and lemmas needed to prove theorem III]; 4) [Proof of Theorem III;] 5) Derivation of theorem I from theorems II and III; References.

Grayev, M.I. Unitary Representations of Real Simple Lie Groups 335

This article was presented at the January 20, 1956 Session of the All-Union Conference on Functional Analysis and its Applications. The article contains the following sections: Introduction; 1) G_{pq} group; parameters and an invariant measure of G_{pq} group;

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2) Generalized linear elements and transitive manifolds; 3) Discrete series of representations of type 1; 4) Irreducibility of representations of a discrete series; 5) Traces of representations of a discrete series; 6) Indiscrete basic series of unitary representations of $G_{p,q}$ group; References.

Muchnik, A.A. Solution of Post's Reducibility Problem and of Certain Other Problems of the Theory of Algorithms. I. Basic results of the article were presented at the October 16, 1956 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; Ch. I. Functional Representation of Partially Recursive Operators; 1) Cortege and quasi-cortege; 2) Functional representations of operators; 3) Universal partially recursive operator; 4) Calculation [solution] of M - [Medevedev] problems; Ch.II. Decision Problems of Enumerable Sets; 1) Semilattices $\mathfrak{A}(p)$; 2) Post's reducibility problem; References. 390

Muchnik, A.A. Isomorphism of Systems of Recursively Enumerable Sets With Effective Properties 407

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The basic results given in this article were presented at the December 17, 1957 session of the Moscow Mathematical Society. The article contains the following sections: 1) Introduction; 2) On the correspondence (reducibility) of systems of sets; 3) Effective inseparability; 4) Quasi-effective properties; References.

Raykov, D.A. Completely Continuous Spectra of Convex Spaces 413
Basic results given in this article were presented at the December 3, 1957 session of the Moscow Mathematical Society. The article contains the following sections: Introduction; 1) Preliminary information and agreements of a general character; 2) Preliminary information on projective limits; 3) Preliminary information on inductive limits; 4) Spaces of type (S); 5) Spaces of type ($\bar{S}$); 6) Spaces of type ($\bar{S}'$); 7) Preliminary information from the theory of duality; 8) Conjugate mappings; 9) Duality of classes ($\bar{S}$) and ($\bar{S}'$); 10) Nondegenerated spectra; References.

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KRASNOSEL'SKIY, M.A.; RUTITSKIY, Ya.B. (Voronezh)

Orlicz' spaces and nonlinear integral equations. Trudy Mosk. mat. ob-va 7:63-120 '58. (MIRA 11:8)
(Functional analysis)

AUTHOR: Kolmogorov, A.N., Krasnosel'skiy, M.A. SOV/42-13-3-12/41

TITLE: Mark Grigor'yevich Kreyn (on the occasion of his 50th birthday) (Mark Grigor'yevich Kreyn (K pyatidesyatiletiyu so dnya rozhdeniya))

PERIODICAL: Uspekhi Matematicheskikh Nauk, 1958, Vol 13, Nr 3, pp 213-224 (USSR)

ABSTRACT: This is a short biography and very detailed appreciation of the mathematical merits of the versatile and extraordinary intensively working mathematician Kreyn. It contains a photo of Kreyn and a chronological list of his scientific publications with 151 numbers (from 1926 to 1958).

Card 1/1

AUTHORS: Krasnosel'skiy,M.A. and Pustyl'nik,Ye.I. SOV/20-122-6-6/49

TITLE: **The Use of Fractional Powers of Operators in the Study of a Fourier Series by the Eigen Functions of Differential** Operators (Ispol'zovaniye drobnykh stepeney operatorov pri izuchenii ryadov Fur'ye po sobstvennym funktsiyam differentsial'nykh operatorov)

PERIODICAL: Doklady Akademii nauk SSSR, 1958,Vol 122,Nr 6,pp 978-981 (USSR)

ABSTRACT: Several questions connected with the theory of Fourier series are formulated and answered by the authors in a very clear manner by the application of negative **fractional powers of** differential operators. 5 theorems are given altogether, e.g.: Theorem: Let T be a positively definite selfadjoint operator in the Hilbert space H, let there exist a completely continuous inverse operator. Let λ_i and u_i be eigenvalues and eigenfunctions of T: $Tu_i = \lambda_i u_i$. Let Ω_α be the region of definition of T^α ($\alpha > 0$).
Let the operator $T^{-\beta}$ be continuous and let it act from H into a space $E \subset H$. Let $f \in \Omega_{\beta+\gamma}$ ($\gamma \geqslant 0$). Then the Fourier series

$$(f,u_1)u_1+(f,u_2)u_2+\dots+(f,u_n)u_n+\dots$$

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The Use of Fractional Powers of Operators in the Study of a Fourier Series by the Eigen Functions of Differential Operators SOV/20-122-6-6/49

converges to f (norm convergence with respect to the norm of E), and we have

$$\left\| f - \sum_{i=1}^{n} (f,u_i)u_i \right\|_E = O(a_n^{-\gamma}),$$

where a_n is the smallest one of the numbers $\lambda_{n+1}, \lambda_{n+2}, \dots$.

ASSOCIATION: Voronezhskiy gosudarstvennyy universitet (Voronezh State University)

PRESENTED: June 5, 1958, by S.L.Sobolev, Academician

SUBMITTED: June 4, 1958

Card 2/2

AUTHORS: Bakhtin, I.A., and Krasnosel'skiy, M.A. SOV/20-123-1-3/56

TITLE: On the Theory of Equations With Concave Operators (K teorii uravneniy s vognutymi operatorami)

PERIODICAL: Doklady Akademii nauk SSSR, 1958, Vol 123, Nr 1, pp 17-20 (USSR)

ABSTRACT: Let K and K_1, $K \subset K_1$, be cones in the real Banach space E. $x \leq y$ denotes that $y-x \in K_1$. For arbitrary $x,y \in K$ let the relation $\|x\| \leq m \|y\|$ follow from $x \leq y$. Let the nonlinear operator A be defined on K and let $AK \subset K$. From $x \leq y$ let follow $Ax \leq Ay$. To every $x \in K$ let exist numbers $\alpha, \beta > 0$ so that $\alpha u_0 \leq Ax \leq \beta u_0$. For $0 < t < 1$ let $Atx \geqslant tAx$; $Atx \neq tAx$ for all $x \in K$ for which $x \geqslant \gamma u_0$, $\gamma > 0$. Operators A with these properties are called concave. The concave operator A is called $\{K, u_0\}$-concave if for $x, y \in K$, where $x \geqslant \gamma u_0$, $y \geqslant \gamma u_0$ $(\gamma > 0)$, $y-x \in K_1$, it holds that from $tx \leq y$ $(tx \neq y,\ t > 0)$ there follows $Ay - tAx \geqslant \delta u_0$, where $\delta > 0$.

Theorem: Let A be concave and completely continuous, let $\varphi = A\varphi$ have a unique, not vanishing solution φ^* in K. Then the successive approximations $\varphi_{n+1} = A\varphi_n$ converge to φ^* with

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respect to the norm for all $\varphi_0 \in K$, $\|\varphi_0\| \neq 0$.

Theorem: Let the functions $K(s,t,u)$ and $\Phi(s,t,u) = \frac{1}{u} K(s,t,u)$ continuous in u and positive for $u > 0$ have the following properties: a) $K(s,t,0) \equiv 0$, $K(s,t,u)$ monotonely increasing for increasing u, $0 \leq u < \infty$; b) for $0 \leq u_1 < u_2$ it holds:

$$\inf_{a \leq s,t \leq b} \left[\Phi(s,t,u_1) - \Phi(s,t,u_2)\right] > 0;$$

c) for $u \to 0$, $u \to \infty$ there exist uniform limit values of $\Phi(s,t,u)$ with respect to s,t; for $u \to 0$ a positive bounded function is obtained, for $u \to \infty$ either a positive bounded function or zero is obtained.

Let $A\varphi = \int_a^b K[s,t,\varphi(t)]dt + f(s)$. Let the equation $\varphi = A\varphi$, where $f(s)$ is a non-negative function, have a positive solution $\varphi^*(s)$.

Then the sequence

$$\varphi_{n+1}(s) = \int_a^b K[s,t,\varphi_n(t)]\,dt + f(s)$$

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converges uniformly to $\varphi^*(s)$ for every non-negative function $\varphi_0(s)$, $\varphi_0(s) \not\equiv 0$.

Two further theorems contain refinements of these assertions for some special cases (e.g. for special $\{K_1, u_0\}$ -concave operators).

There are 8 Soviet references.

ASSOCIATION: Voronezhskiy gosudarstvennyy universitet (Voronezh State University)

PRESENTED: June 9, 1958, by P.S.Aleksandrov, Academician

SUBMITTED: May 10, 1958

AUTHORS: Krasnosel'skiy, M.A. and Perov, A.I. SOV/20-123-2-6/50

TITLE: On a Principle of the Existence of Bounded, Periodic and Almost-Periodic Solutions of a System of Ordinary Differential Equations (Ob odnom printsipe sushchestvovaniya ogranichennykh, periodicheskikh i pochti-periodicheskikh resheniy u sistemy obyknovennykh differentsial'nykh uravneniy)

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ABSTRACT: Given the system

$$(1) \qquad \frac{dx}{dt} = f(t,x),$$

where $x = (x_1,\dots,x_n)$ and $f = (f_1,\dots,f_n)$, $f_i = f_i(t,x_1,\dots,x_n)$ and the f_i are continuous in $-\infty < t, x_1,\dots,x_n < +\infty$. Let further $\lambda(x)$ and $\mu(x)$ be two continuously differentiable functions, $\lambda(-x) = \lambda(x)$,

$$(f(t,x), \operatorname{grad} \lambda(x)) = \sum_{i=1}^{n} f_i \frac{\partial \lambda}{\partial x_i} > 0$$

for $\|x\| \geq R > 0$. Let $m = \min\limits_{\|x\|=R} \lambda(x)$, $M = \max\limits_{\|x\|=R} \lambda(x)$. On the set T of those $x \in E^n$ for which $m \leq \lambda(x) \leq M$, $\|x\| > R$, let $\mu(x)$ satisfy

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the condition

$$(f(t,x),\mathrm{grad}\,\lambda(x)+\mathrm{grad}\,\mu(x)) = \sum_{i=1}^{n} f_i \left(\frac{\partial \lambda}{\partial x_i} + \frac{\partial \mu}{\partial x_i} \right) \geqslant 0,$$

where $\lim\limits_{x\in T,\|x\|\to +\infty} |\mu(x)| = +\infty$.

Theorem: Under the given assumptions (1) has at least one uniformly bounded solution on $(-\infty,\infty)$. If the f_i are periodic in t, then (1) has at least one periodic solution of the same period. If the f_i are almost-periodic (uniformly in every sphere) in t, then (1) has at least one almost-periodic solution. The theorem holds in a strengthened form if instead of $\lambda(-x) = \lambda(x)$ it is assumed that outside of a certain sphere $\mathrm{grad}\,\lambda(x) \neq 0$ and that the field of $\mathrm{grad}\,\lambda(x)$ has a nonvanishing rotation on spheres of a sufficiently large radius (see [Ref 1, 7]). The proof of the theorems is based on the theorem: On the boundary Γ of a bounded domain $G \subset E^n$ let the vector fields $f(t,x)$, $-\infty<t<\infty$ have a rotation different from zero. For (1)

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